AN ACCURATE DETERMINATION OF THE CHARACTERISTIC IMPEDANCE MATRIX OF COUPLED SYMMETRICAL LINES ON CHIPS BASED ON HIGH FREQUENCY S-PARAMETER MEASUREMENTS

Thomas-Michael Winkel*, Lohit Sagar Dutta**, Hartmut Grabinski**

*IBM, Schönaicher Str. 220, 71032 Böblingen, Germany,
**University of Hannover, Schneiderberg 32, 30167 Hannover, Germany

ABSTRACT

A new method has been developed to determine the characteristic impedance matrix of a symmetric coupled lossy two line system on chips. The presented results are based on high frequency measurements of the scattering parameters. A comparison between the measured and analytical calculated results is given and shows excellent agreement.

INTRODUCTION

Interconnects increasingly dominate the performance of advanced integrated circuits due to the rising clock rates, decreasing line widths and spacings and increasing line lengths. A line system is completely characterized e.g. by its propagation constant in combination with its characteristic impedance. Therfore, methods are needed to determine these characteristic parameters from high frequency measurements. An exact experimental determination of the propagation constant can be performed by using the method described in [1]. Up to now, the high accurate calculation of the characteristic impedance from high frequency S-parameter measurements is only known for single lines ([2] - [5]) while the determination of the characteristic impedance matrix for coupled lines is still problematic. The aim of this work is to present a complete and very accurate characterization of coupled symmetrical lines based on high frequency measurements. The presented method for the determination of the characteristic impedance matrix is an extention of the method proposed in [5] for single lines.

MEASUREMENT SETUP

The S-parameter measurements were performed with a HP 8720B 2 port network analyzer in a frequency range from 200 MHz to 20 GHz. The network analyzer was connected through special flexible microwave cable to cascade coplanar microwave probes with a characteristic impedance of 50 Ω. The off-chip calibration of the network analyzer, the microwave cable and the microwave probes were performed by using the short-open-load-through (SOLT) calibration procedure as was proposed by Cascade Microtech for the HP 8720B network analyzer [6]. The reference standards used for the calibration were fabricated by cascade microtech on special alumina substrate. As a result of the calibration procedure, the reference planes of the measurements had to be taken at the on chip contact points of the microwave probes (contact pads in figure 1).

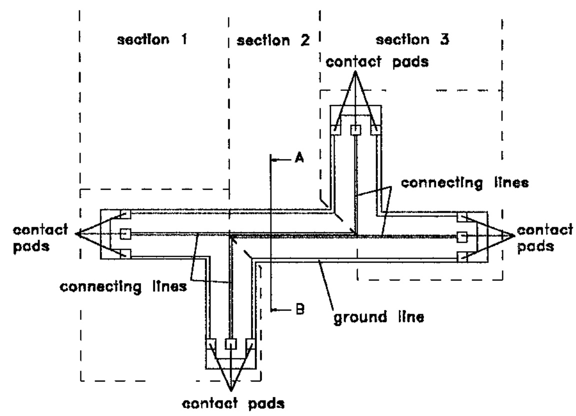

Figure 1: Geometry of the investigated coupled lines

The thickness of the silicon substrate was taken to be 500 μm, the oxid thickness 520 nm, and the metal layer thickness (aluminium with 1% silicon) 520 nm.

To ensure a well defined potential and hence also a well specified wave propagation along the investigated lines, the single as well as the coupled signal lines were surrounded by ground lines as shown in figure 1. The ground lines were electrically connected to the silicon substrate. Due to the large distance of approximatly 100 μm existing between the signal and ground lines it is ensured that the behavior of the investigated lines corresponds more to real interconnects on integrated circuits than to coplanar lines.

0-7803-4603-6/97/$5.00 (c) IEEE

THEORY

The chain parameter matrix A_c of a segment of the coupled lines is given by:

$$A_c = \begin{pmatrix} \cosh[\gamma_c \cdot l_c] & Z_c \cdot \sinh[\gamma_c \cdot l_c] \\ Z_c^{-1} \cdot \sinh[\gamma_c \cdot l_c] & \cosh[\gamma_c \cdot l_c] \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \quad (1)$$

γ_c denotes the 2×2 matrix of the propagation constant, Z_c the characteristic impedance matrix (2×2) and l_c the length of the coupled line segment. If the chain parameter matrix A_c in equation (1) is given, Z_c can be calculated from:

$$Z_c^2 = A_{12} \cdot A_{21}^{-1} \quad (2)$$

In order to calculate the matrix elements Z_{c11} and Z_{c12} from Z_c one has to first transform Z_c^2 into the diagonal form:

$$T_U \cdot Z_c^2 \cdot T_U^{-1} = \begin{pmatrix} Z_{ce}^2 & 0 \\ 0 & Z_{co}^2 \end{pmatrix} \quad (3)$$

with

$$Z_{ce} = Z_{c11} + Z_{c12}$$
$$Z_{co} = Z_{c11} - Z_{c12} \quad (4a,b)$$

and the transformation matrix

$$T_U = \frac{1}{\sqrt{2}} \cdot \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad (5)$$

In general, the chain parameter matrix of a coupled line segment, as given by equation (1), cannot directly be measured since the coupled line segments are embedded in appropriate contact structures and connecting lines as shown in figure 1. Therefore, the coupled line segment, which can be represented by its scattering chain parameter matrix T_L, is embedded in unknown error networks which can be formally described by the 4×4 scattering chain parameter matrices T_A and T_B. The measured scattering chain parameter matrices T_{m1} and T_{m2} of two complete structures with different coupling line lengths (l_1 and l_2) are therfore given by:

$$T_{m1} = T_A \cdot T_{L_1} \cdot T_B \quad (6)$$

$$T_{m2} = T_A \cdot T_{L_2} \cdot T_B \quad (7)$$

In order to eleminate e.g. the error network T_B one has to first calculate the inverse matrix of T_{m1} and then multiply the result from the left with T_{m2}:

$$T_{m2} \cdot T_{m1}^{-1} = T_A \cdot T_{L_2} \cdot T_B \cdot T_B^{-1} \cdot T_{L_1}^{-1} \cdot T_A^{-1} = T_A \cdot T_{L_2} \cdot T_{L_1}^{-1} \cdot T_A^{-1} \quad (8)$$

If one now calculates the measured chain parameter matrix A_m' from the <u>left</u> hand side of equation (8) with the help of a known matrix transformation (T-parameter to A-parameter) one can then calculate the impedance matrix Z_{ms} from A_m', which we define as the "measured substitute impedance matrix", by (c.f. equation (1) and (2)):

$$Z_{ms}^2 = A_{m12}' \cdot A_{m21}^{-1\prime} \quad (9)$$

Using the transformation matrix T_u one can diagonalize Z_{ms}^2 (c.f. equation (3)). In Z_{ms} the influences of the contact pads and the connecting lines are still included. In order to calculate the true characteristic impedance matrix Z_c, one has to first characterize the scattering chain parameter matrix T_A of the error network which describes only the line segments as depicted in figure 2.

The chain parameter matrices of the line segments are represented by A_o for the open pad lines, A_p for the pad line segments, A_s for the single line segments connecting the pads to the coupled lines and A_c for the coupled line segment. A_o, A_p, A_s and A_c are given by:

$$A_o = \begin{pmatrix} 1 & 0 \\ \frac{\tanh[\gamma_p \cdot l_o]}{Z_p} & 1 \end{pmatrix} ;$$

$$A_p = \begin{pmatrix} \cosh[\gamma_p \cdot l_p] & Z_p \cdot \sinh[\gamma_p \cdot l_p] \\ \frac{\sinh[\gamma_p \cdot l_p]}{Z_p} & \cosh[\gamma_p \cdot l_p] \end{pmatrix} ;$$

$$A_s = \begin{pmatrix} \cosh[\gamma_s \cdot l_s] & Z_s \cdot \sinh[\gamma_s \cdot l_s] \\ \frac{\sinh[\gamma_s \cdot l_s]}{Z_s} & \cosh[\gamma_s \cdot l_s] \end{pmatrix} ; \quad (10)$$

$$A_c = \begin{pmatrix} \cosh[\gamma_c \cdot l_c] & Z_c \cdot \sinh[\gamma_c \cdot l_c] \\ Z_c^{-1} \cdot \sinh[\gamma_c \cdot l_c] & \cosh[\gamma_c \cdot l_c] \end{pmatrix}$$

In equation (10) γ_s is the propagation constants of a single line (with line width w) and γ_p of a line having the same width as the pads. Z_s and Z_p are the characteristic impedances of these two lines respectively. γ_c and Z_c are the matrices of the propagation constant and the characteristic impedance of the coupled line segment. l_o, l_p, l_s and l_c are the line lengths as depicted in figure 2. The propagation constants γ_s and γ_p as well as the characteristic impedances Z_s and Z_p can be calculated from additional measurements on four single lines and a seperate pad structure as described in [5].

The unknown matrix T_A can thus analytically be calculated through a simple transformation (A- to T-parameter transformation) from the normalized 4×4 chain parameter matrix A':

$$A' = \begin{pmatrix} Z_0^{-\frac{1}{2}} & 0 \\ 0 & Z_0^{\frac{1}{2}} \end{pmatrix} \cdot A \cdot \begin{pmatrix} Z_c^{\frac{1}{2}} & 0 \\ 0 & Z_c^{-\frac{1}{2}} \end{pmatrix} \tag{11}$$

with

$$Z_0 = \begin{pmatrix} Z_0 & 0 \\ 0 & Z_0 \end{pmatrix} \tag{12}$$

where Z_0 is the reference impedance of the microwave probes, in our case 50 Ω. By choosing the line length of the coupled line segment $l_c = 0$ the chain parameter matrix A (equation (11)) can be expressed by:

$$A_o \cdot A_p \cdot A_s \rightarrow A = \begin{pmatrix} a_{11} & 0 & a_{13} & 0 \\ 0 & a_{11} & 0 & a_{13} \\ a_{31} & 0 & a_{33} & 0 \\ 0 & a_{31} & 0 & a_{33} \end{pmatrix} \tag{13}$$

With respect to equation (3) the matrices $Z_c^{1/2}$ and $Z_c^{-1/2}$ in equation (11) can be given by:

$$Z_c^{\frac{1}{2}} = \begin{pmatrix} \sqrt{Z_{ce}} & 0 \\ 0 & \sqrt{Z_{co}} \end{pmatrix} ; \quad Z_c^{-\frac{1}{2}} = \begin{pmatrix} \frac{1}{\sqrt{Z_{ce}}} & 0 \\ 0 & \frac{1}{\sqrt{Z_{co}}} \end{pmatrix} \tag{14}$$

After an A-T parameter transformation one can now calculate the measured substitute impedance matrix in equation (9) analytically from the right hand side of equation (8). After equating the measured substitute impedance matrix as calculated from the left hand side of equation (8) to the measured substitute impedance which was calculated analytically from the right hand side of equation (8) one can express finally the elements of the diagonalized characteristic impedance matrix by:

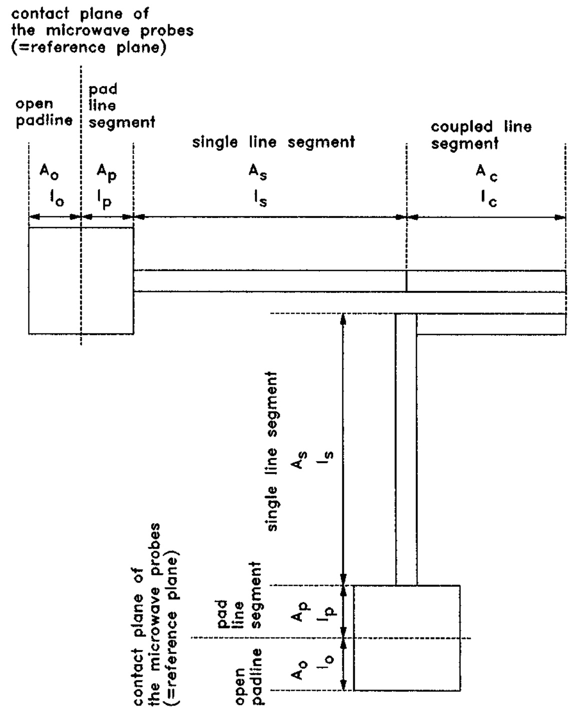

Figure 2: Subdivided signal lines of the error network T_A

$$Z_{ce} = \sqrt{\frac{a_{33}^2 \cdot Z_0^2 \cdot Z_{mse}^2 + a_{13}^2}{a_{31}^2 \cdot Z_0^2 \cdot Z_{mse}^2 + a_{11}^2}} \tag{15}$$

and

$$Z_{co} = \sqrt{\frac{a_{33}^2 \cdot Z_0^2 \cdot Z_{mso}^2 + a_{13}^2}{a_{31}^2 \cdot Z_0^2 \cdot Z_{mso}^2 + a_{11}^2}} \tag{16}$$

MEASUREMENT RESULTS

In order to compare the characteristic impedances as derived from measurements with analytically calculated values, the **R**, **L**, **C** and **G** matrices had to be first calculated, e.g. by the analytical method as proposed in [7]. The characteristic impedance matrix were then derived as follows:

$$Z_c^2 = (R + j\omega L) \cdot (G + j\omega C)^{-1} \tag{17}$$

The results presented in figure 3 and 4 were obtained from measurements carried out on coupled lines with a line width of $w = 5\mu m$ and spacing of $s = 5\mu m$ on a silicon substrate with a 0.015 Ωcm resistivity. The

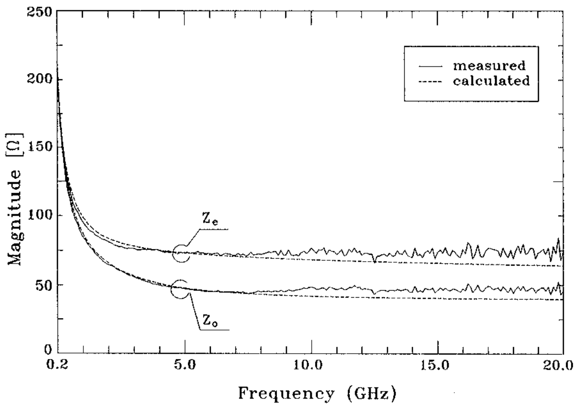

Figure 3: Measured and calculated characteristic impedances Z_e and Z_o of a coupled line (line width w = 5 μm, spacing s = 5 μm)

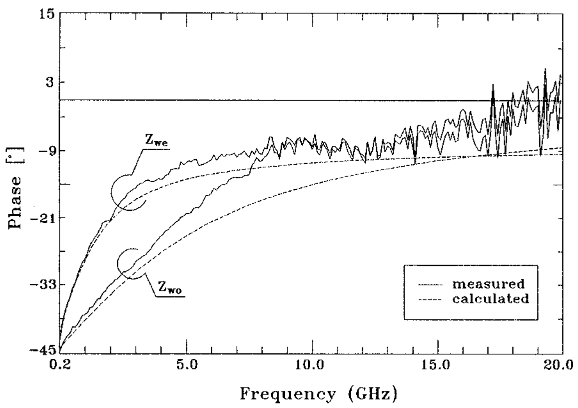

Figure 4: Measured and calculated characteristic impedances Z_e and Z_o of a coupled line (line width w = 5 μm, spacing s = 5 μm)

measured magnitudes and phases of the characteristic impedances in the even- and odd-mode are compared to the analytically calculated values (equation (17)). As depicted in figure 3 and 4 the measured and calculated values show a very good agreement for the magnitudes and phases in the even- and odd-mode. The differences that can be observed especially in figure 4 for frequencies > 5 GHz is a result of the usage of uncertain input values for the calculations. Due to the tolerances in the nominal values, e.g. in line widths and thickness as well as in the oxid thickness resulting from the etching processes, these values cannot be expressed exactly.

CONCLUSION

Knowledge over the characteristic impedance matrix and the matrix of the propagation constant is essential when an experimental investigation on coupling effects and signal behavior of interconnects on semiconductor substrates has to be carried out. In this paper a novel method for the determination of the characteristic impedance of lossy lines on semiconductor substrates based on high frequency S-parameter measurements has been presented and shows a very good agreement with analytical calculated results in a wide frequency range. Further work has been carried out to determine the influence of different substrates on characteristic parameters of lines.

REFERENCES

[1] Thomas-Michael Winkel, L. S. Dutta, H. Grabinski, E. Grotelüschen, "Determination of the Propagation Constant of Coupled Lines on Chips Based on High Frequency Measurements", *IEEE Multi Chip Module Conference 1996 MCMC'96*, pp. 99-104, February 1996

[2] W.R. Eisenstadt, Y. Eo, "S-Parameter-Based IC Interconnection Transmission Line Characterization", *IEEE Transactions on Components, Hybrids and Manufactoring Technology*, Vol. 15, No. 4, pp. 483-490, August 1992

[3] Dylan F. Williams, Roger B. Marks, "Accurate Transmission Line Characterization", *IEEE Microwave and Guided Wave Letters*, Vol. 3, No. 8, pp. 247-249, August 1993

[4] Stefan Zaage, "Meßtechnische Charakterisierung des Breitband-Übertragungsverhaltens von Leitungen auf Silizium-Substraten", Dissertation, Fakultät für Maschinenwesen, Universität Hannover, August 1993

[5] Thomas-Michael Winkel, Lohit Sagar Dutta, Hartmut Grabinski, "An Accurate Determination of the Characteristic Impedance of Lossy Lines on Chips Based on High Frequency S-Parameter Measurements", *IEEE Multi-Chip Module Conference MCMC'96*, pp. 190-195, February 1996

[6] Cascade Microtech, "Microwave Wafer Probe Calibration Constants", *Instruction Manual*

[7] E. Grotelüschen, L.S. Dutta, S. Zaage, "Full-wave analysis and analytical formulas for the line parameters of transmission lines on semiconductor substrates", *Integration the VLSI journal*, Vol. 16, pp. 33-58, 1993

0-7803-4603-6/97/$5.00 (c) IEEE